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NO. 1014

FORMULAE FOR MASS MEDIAN AND MASS MEAN DROP DIAMETERS (U)

by

J. Monaghan, G.A. Hill and W.G. Soucey

PCN No. 13E10

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FORMULAE FOR MASS MEDIAN AND MASS MEAN DROP DIAMETERS (U)

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by

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J. Monaghan, G.A. Hill and W.G. Soucey

ABSTRACT

Formulae are derived to calculate the mass median (D_0) and mass mean (D_M) diameters of droplets in sprays. These formulae are based on an inverse exponential relationship which is empirically observed to occur in liquid sprays. The median diameter divides the total mass of liquid in half while the mean diameter represents a more commonly used statistical average for identifying random samples. The formulae are used to describe the characteristics of a typical set of experimental drop data and are compared to conventional formulae for calculating these parameters.

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FORMULAE FOR MASS MEDIAN AND MASS MEAN DROP DIAMETERS (U)

by

J. Monaghan, G.A. Hill and W.G. Soucey

INTRODUCTION

When describing the characteristics of a liquid spray, two of the more commonly used parameters are the mass median (D_O) and mass mean (D_M) diameters of the droplets. The mass median diameter represents the droplet diameter at which half the mass of liquid is above and half the mass of liquid is below. However, because the mass is related to the cube of the diameter, it is found that the mass median usually lies close to the largest sized drops observed on a trial. The mass mean diameter, on the other hand, represents the value for which the sum of the deviations in mass diameter from it is zero. This is a more commonly used type of averaging term and for drop diameters does not rest as close to the largest drop as does the mass median diameter.

The model used here depends upon an exponential relationship between the cumulative number of drops and the drop diameter. This relationship has been empirically observed to exist for many different

spraying systems. The model is used to determine both D_0 and D_M for a liquid spray generated by an explosive dissemination and the results are compared to the standard methods for calculating these parameters.

THEORY

The standard method for determining the mass mean diameter or mass median diameter is simply taken to be the common statistical methods for calculating means or medians (1,2). Therefore, the following formulae are presented with no derivations:

$$D_0 = D \text{ when } m = M_T/2.0 \tag{1}$$

and

$$D_M = \sqrt[3]{\frac{\Sigma n_i \quad D_i{}^3}{N_T}} \tag{2}$$

where
D = arbitrary drop diameter (mm)
m = arbitrary drop mass (g)
M_T = total mass of all sampled drops (g)
n_i = number of drops of diameter D_i
N_T = total number of sampled drops

To derive the equations for the modified method of evaluating D_0 and D_M, the drop numbers and drop mass in each diameter class are summed from the maximum diameter to the smaller diameters using the following definitions:

$$N_k = \sum_{i = n_{max}}^{k} n_i \tag{3}$$

$$M_k = \sum_{i = m_{max}}^{k} m_i \tag{4}$$

where m_i = mass of drops at diameter D_i (g)

The observed drop number-diameter distribution (N vs D) empirically always appears to be well represented by one of the following two equations (these equations are similar to those of Rosin and Ramler, Reference 3):

$$\ln N = a - bD \tag{5}$$

$$\ln N = a - bD^2 \tag{6}$$

where a is the intercept at the ordinate

e^a is the total number of drops, N_T

b is the absolute value of the slope of the line.

For each drop class, assuming that the drops are spherical, it is clear that:

$$m_k = \frac{\pi\rho}{6} D_k{}^3 n_k \tag{7}$$

However, Equation 3 and 4 imply that $m_k = \Delta M_k$ and $n_k = \Delta N_k$ which when substituted in Equation 7 give:

$$\Delta M_k = \frac{\pi\rho}{6} D_k{}^3 \Delta N_k \tag{8}$$

Allowing the class interval to approach zero, we obtain the differential equation corresponding to Equation (8):

$$dM = \frac{\pi\rho}{6} D^3 dN \tag{9}$$

In order to eliminate N in Equation 9, Equation 5 is manipulated to give:

$$dN = -be^{a-bD}dD \tag{10}$$

Substituting 10 in 9 gives:

$$dM = \frac{-\pi\rho b e^a}{6} D^3 e^{-bD} dD \tag{11}$$

Integration of Equation 11 is performed by reversing the order of integration (the order of summation as defined in Equation 4) to give:

$$M_k = \frac{\pi\rho b e^a}{6} \int_{D_k}^{\infty} D^3 e^{-bD} dD \qquad (12)$$

or: $$M_k = \frac{\pi\rho N_T}{6b^3} \left\{ \int_0^{\infty} (bD)^3 e^{-bD} d(bD) - \int_0^{D_k} (bD)^3 e^{-bD} d(bD) \right\} \qquad (13)$$

The first integral is the gamma function of 4, $\Gamma 4$. Solutions of this integral are listed in many handbooks (Reference 4 for instance). The total mass is determined when $D_k \rightarrow 0$ or:

$$M_T = \frac{\pi\rho N_T}{6b^3} \Gamma 4 = \frac{\pi\rho N_T}{b^3} \qquad (14)$$

Also: $$M_k = M_T \left[1 - I(bD_k, 3)\right] \qquad (15)$$

where: $$I(bD_k, 3) = \frac{\int_0^{D_k} (bD)^3 e^{-bD} d(bD)}{\int_0^{\infty} (bD)^3 e^{-bD} d(bD)} \qquad (16)$$

values of which are listed by Pearson (5).

The mass mean diameter can be alternatively stated as:

$$D_M = \sqrt[3]{\frac{6}{\pi\rho} \frac{M_T}{N_T}} \qquad (17)$$

Substituting 14 into 17, a new expression for D_M is obtained:

$$D_M = \sqrt[3]{6/b^3} = 1.817/b \qquad (18)$$

The mass median diameter, D_0 is obtained from Equation (15) when:

$$M_0/M_T = 0.5 = 1 - I(bD_0, 3) \qquad (19)$$

Using Pearson's Tables (5), the following values are obtained:

$$bD_0 = 3.671$$

or $$D_0 = 3.671/b \qquad (20)$$

Returning to Equation (10) and using Equation (6) instead of (5), gives:

$$dN = -2bDe^{a-bD^2} dD \qquad (21)$$

Substituting 21 into 9 and reversing the order of integration as before, yields:

$$M_k = \frac{\pi\rho e^a b}{3} \int_{D_k}^{\infty} D^4 e^{-bD^2} dD \tag{22}$$

or: $$M_k = \frac{\pi\rho N_T}{6b^{1.5}} \left\{ \int_0^{\infty} (bD^2)^{1.5} e^{-bD^2} d(bD^2) - \int_0^{D_k} (bD^2)^{1.5} e^{-bD^2} d(bD^2) \right\} \tag{23}$$

Proceeding as before:

$$M_T = \frac{\pi\rho N_T}{6b^{1.5}} \Gamma 2.5 = \frac{0.2216\pi\rho N_T}{b^{1.5}} \tag{24}$$

$$M_k = M_T \left[1 - I(bD_k^2, 1.5)\right] \tag{25}$$

$$D_M = \sqrt[3]{\frac{1.329}{b^{1.5}}} = \frac{1.0995}{b^{0.5}} \tag{26}$$

$$D_0 = I(bD_0^2, 1.5) = 0.5 \tag{27}$$

which using Reference 5 yields:

$$D_0 = 1.477/\sqrt{b} \tag{28}$$

Equations 18 and 26, 20 and 28 represent the mass mean and mass median diameters that were sought. It can be seen that these are determined simply from the slope of the line, b, in Equations 5 and 6 representing the number-diameter distributions. In practice, logarithms to the base 10 are often used to evaluate b. The slope obtained is of course less than when natural logs are used by a factor of 2.3026 and the equations become:

$$D_M = -0.7890/b \tag{29}$$

$$D_0 = -1.594/b \tag{30}$$

for $\ln N \propto D$

and

$$D_M = 0.7245/\sqrt{b} \tag{31}$$

$$D_0 = 0.9733/\sqrt{b} \tag{32}$$

for $\ln N \propto D^2$

It is interesting to note that in both cases, the mass mean diameter is

simply a constant fraction times the mass median diameter (0.495 and 0.744). This results from Equations 5 and 6 where all the drops are related to all the diameters by one equation.

EXPERIMENTAL

To verify these expressions, experimental data were obtained from a recent spray trial. In this trial, liquid droplets were generated by an explosive dissemination. The droplets were released at 9 meters elevation and were collected downwind on 3-way detector paper (NSN 6665-21-845-8613). The sizes of stains were related to the corresponding drop diameters by a previous calibration procedure not discussed here. Table 1 lists the drop diameters measured and the number of drops of each size.

RESULTS AND DISCUSSION

The experimental data were fed directly to the computer and analyzed by the computer programs listed in Appendix A. Table 1 lists the reduced data required for evaluating the averaged diameters as given by Equations 1, 2, 29, 30, 31 and 32.

The mass median and mass mean diameters are listed in Table 2 for each of the techniques. Figures 1 and 2 show plots of the cumulative frequency versus the drop diameter and drop diameter squared respectively. It can be seen from the figures that the exponential relationship between the cumulative number and drop diameter is a much better approximation to this experimental data than cumulative number vs drop diameter squared. This is quantitatively expressed by a significantly better regression coefficient of 0.990 vs 0.971 respectively.

Comparing the averaged diameter as calculated by Log N vs D and the standard method, it may be seen that the mass mean diameters are very close (0.386 vs 0.385 respectively). However the mass median diameters are significantly different (0.781 vs 0.663 respectively). This is most likely due to the effect of large diameter drops on this parameter. The large diameter drops contain so much of the liquid that only about 3% to 5% of all the drops lie above this parameter. Because they are so few in number, large sampling errors are likely to occur and the standard method considers only those that are sampled. By smoothing the data with a straight line, however this problem is circumvented and the slope technique presented by this work is a much better method of evaluating mass mean or mass median diameters than the standard summation method.

CONCLUSIONS

An improved method has been explained for evaluating the mass mean and mass median diameters of liquid sprays. The standard summation method has been compared to this technique using experimental data from a typical spray trial. The technique was at least as simple as the standard method and was more accurate for the data tested.

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3. Rosin, Pand Rammler, E., J. Inst. Fuel, 7, 29 (1933).
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5. Pearson, K., Tables of the Incomplete Γ-Function, Cambridge University Press, 1934.

TABLE 1: TYPICAL SPRAY TRIAL DATA

Drop Diameter, D (mm)	No. of Drops n	Accumulated Number, N	D^2 $(mm)^2$	D^3 $(mm)^3$	nD^3 $(mm)^3$
1.165	1	1	1.357	1.581	0.104
0.993	2	3	0.987	0.981	1.961
0.917	1	4	0.841	0.772	0.772
0.898	1	5	0.807	0.725	0.725
0.803	1	6	0.645	0.518	0.518
0.784	1	7	0.614	0.482	0.482
0.765	2	9	0.585	0.447	0.895
0.708	2	11	0.501	0.354	0.709
0.689	2	13	0.474	0.326	0.653
0.669	6	19	0.448	0.300	1.800
0.631	4	23	0.399	0.252	1.007
0.612	7	30	0.375	0.230	1.607
0.593	1	31	0.352	0.209	0.209
0.574	2	33	0.330	0.189	0.378
0.555	3	36	0.308	0.171	0.513
0.536	1	37	0.287	0.154	0.154
0.517	3	40	0.267	0.138	0.414
0.498	4	44	0.248	0.123	0.493
0.479	2	46	0.229	0.110	0.219
0.421	2	48	0.178	0.0748	0.150
0.383	10	58	0.147	0.0563	0.563
0.345	1	59	0.119	0.0411	0.0411
0.326	6	65	0.106	0.0346	0.208
0.307	8	73	0.0941	0.0289	0.231
0.288	11	84	0.0827	0.0238	0.262
0.269	5	89	0.0721	0.0194	0.0968
0.249	5	94	0.0622	0.0155	0.0776
0.230	10	104	0.0530	0.0122	0.122
0.211	11	115	0.0446	0.00941	0.104

TABLE 1 (Continued)

Drop Diameter, D (mm)	No. of Drops n	Accumulated Number, N	D^2 $(mm)^2$	D^3 $(mm)^3$	nD^3 $(mm)^3$
0.192	10	125	0.0369	0.00708	0.0708
0.173	12	137	0.0299	0.00516	0.0620
0.154	11	148	0.0236	0.00363	0.0399
0.135	16	164	0.0181	0.00244	0.0390
0.115	18	182	0.0133	0.00154	0.0276
0.0962	38	220	0.00925	8.90×10^{-4}	0.0338
0.0770	37	257	0.00593	4.57×10^{-4}	0.0169
0.0578	27	284	0.00334	1.93×10^{-4}	0.00522
0.0386	18	302	0.00149	5.74×10^{-5}	0.00103

TABLE 2: COMPARISON OF AVERAGED DIAMETERS

Technique	Mass Mean Diam (mm)	Mass Median Diam (mm)	Regression Coef † or Std Dev (±mm)††
Standard	0.385	0.663	± 1.255 ††
Log N vs D	0.386	0.781	0.990 †
Log n vs D^2	0.534	0.718	0.971 †

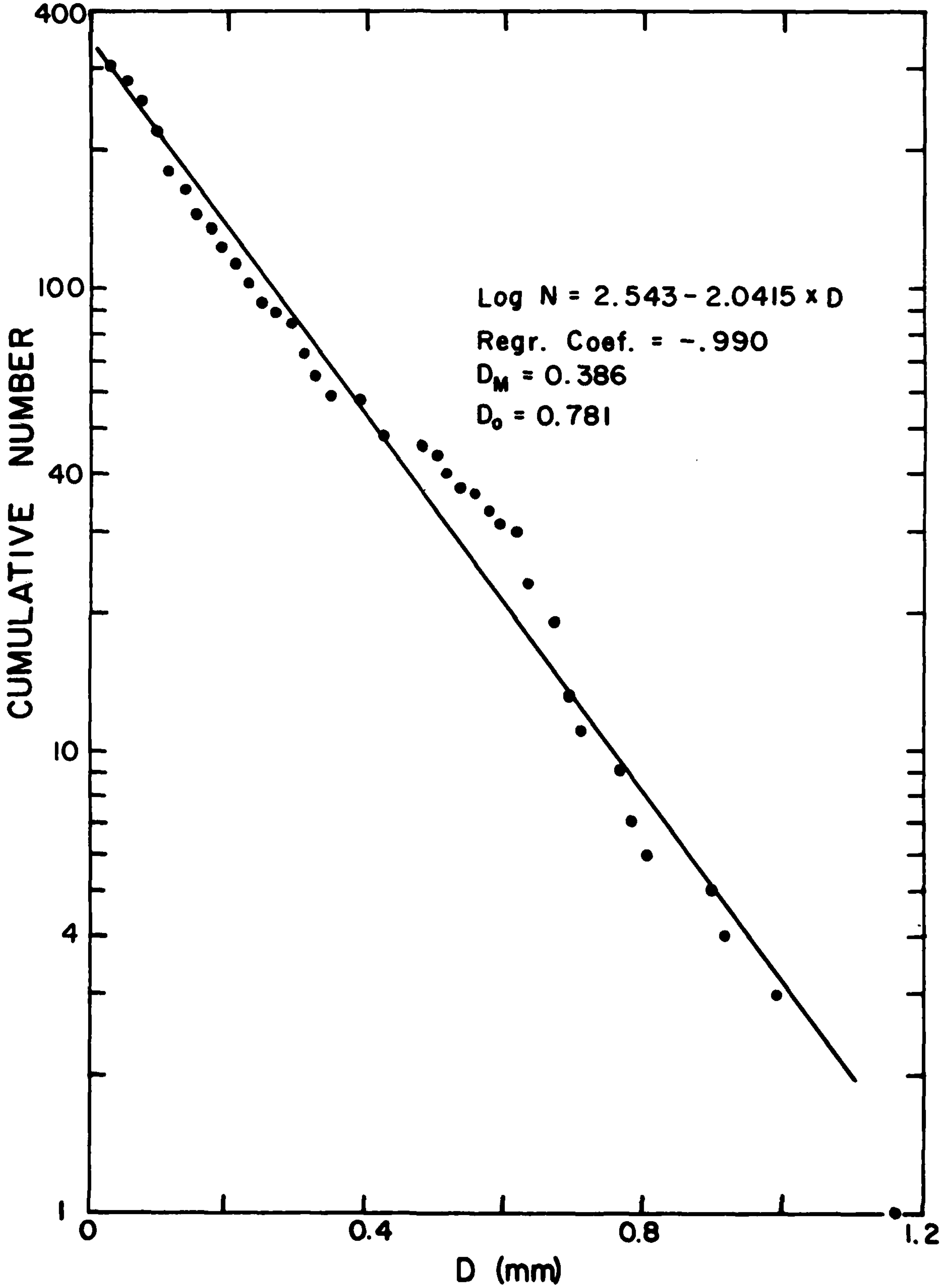

FIGURE 1: Evaluation of Average Diameters from Log N vs D.

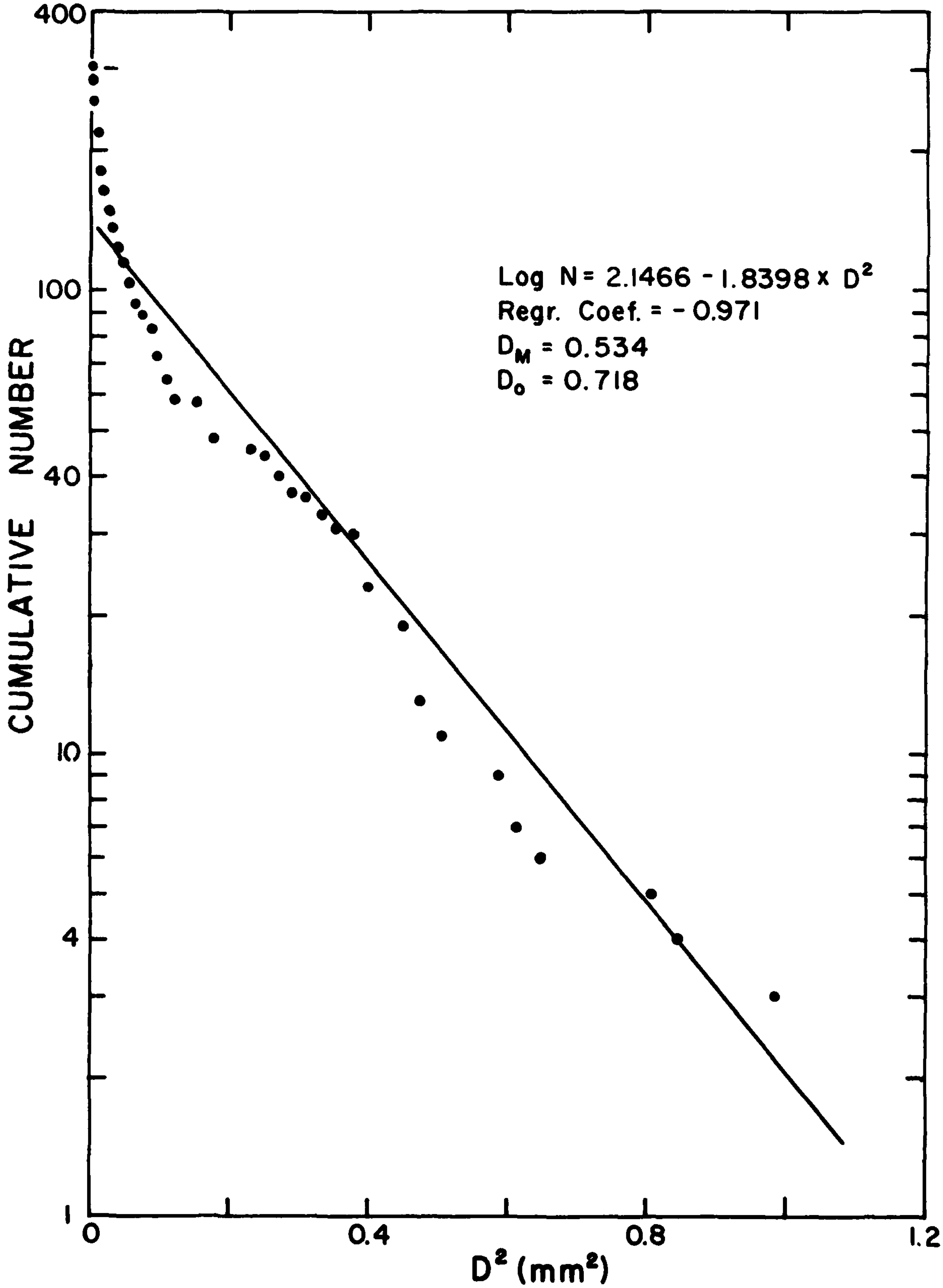

FIGURE 2: Evaluation of Average Diameters from Log N vs D^2.

APPENDIX A

Computer Program for Averaging Drop Diameters

```
45 A(1,M+1)=A(1,M)
   A(2,M+1)=A(2,M)
   IF(M.EQ.I)  GO TO 47
   M=M-1
   GO TO 45
47 A(1,I)=TEMP1
   A(2,I)=TEMP2
   GO TO 25
40 A(2,I)=A(2,I)+A(2,J)
   A(2,J)=0.0
   GO TO 25
30 CONTINUE
25 CONTINUE
   IK=0
   DO 50 J=1,N
   IK=IK+1
   IF(A(2,J).EQ.0.0) GO TO 60
   A(1,IK)=A(1,J)
   A(2,IK)=A(2,J)
   GO TO 50
60 IK=IK-1
50 CONTINUE
   N=IK
C
C     EXPANDING THE MATRIX
C
   SUM=0.0
   SUM1=0.0
   SUM2=0.0
   DO 70 J=1,N
   A(3,J)=0.192*(A(1,J))**0.997
   A(6,J)=A(3,J)**3
   SUM=SUM+A(2,J)
   A(4,J)=A(2,J)*A(3,J)**3
   SUM1=SUM1+A(4,J)
   A(5,J)=SUM1
70 SUM2=SUM2+A(2,J)*A(6,J)**2
   WRITE(6,4) N
   WRITE(6,5) (A(1,J),A(2,J),A(3,J),A(6,J),A(4,J),J=1,N)
C
C     CALCULATE MASS MEAN AND ST. DEV.
C
   SMEAN=(SUM1/SUM)**0.33333
   AVMAS=SUM1/2.0
   AVMASS=3.14156/6.0*AVMAS
   STDEV=(SQRT(SUM2/SUM-(SUM1/SUM)**2))**0.333333
   SUM3=0.0
   DO 100 J=1,N
```

```
//AVDROP    JOB (                    ),'GORDON HILL',
// MSGLEVEL=1,MSGCLASS=A,CLASS=K,TIME=(0,29)
//*MAIN LINES=10,CARDS=20,ORG=RM028
//*FORMAT PU,DDNAME=,DEST=LOCAL
// EXEC FORTXCG
//SYSPRINT DD SYSOUT=A
//SYSTEM DD SYSOUT=A
//FORT.SYSIN DD *
C
C      PROGRAM TO EVALUATE  VMD'S
C
       DIMENSION A(6,1001),CIRCLE(10)
     1 FORMAT(2F10.3)
     2 FORMAT('1     INPUT DATA FOR POSITIONS= ',10A1,' WHICH HAS ',
      1I4,' INPUT CARDS',//,8X,'STAIN SIZE',5X,'NO. OF DROPS',//)
     3 FORMAT(7X,F10.3,5X,F10.1)
     4 FORMAT('1  REDUCED DATA HAS ',I4,' SETS OF DATA',//,8X,
      1'STAIN SIZE',5X,'NO. OF DROPS',5X,'DROP SIZE',8X,'D**3',
      213X,'ND**3',//)
     5 FORMAT(7X,F10.3,5X,F10.1,5X,F10.5,5X,E13.6,5X,E13.6)
     6 FORMAT(0,' SOMETHING WRONG  THE AVERAGE MASS= ',F10.5,' IS BIGGER
      1THAN THE TOTAL MASS=' ,F10.5)
     7 FORMAT('1',5X,' HERE ARE THE AVERAGED DATA',//)
     8 FORMAT(10A1)
     9 FORMAT(5X,' MEAN=',F10.4,/,6X,'STANDARD DEVIATION=',F10.6,/,
      16X,'AVERAGE MASS= ',F10.4,/,6X,'MASS MEDIAN=',F10.4)
       DO 998 NJK=1,2
       READ(5,8) (CIRCLE(I),I=1,10)
       DO 10 J=1,1001
       N=J-1
       READ(5,1)A(1,J),A(2,J)
       IF(A(1,J).EQ.0.0.AND.A(2,J).EQ.0.0)GO TO 20
    10 CONTINUE
    20 WRITE(6,2) (CIRCLE(I),I=1,10),N
       WRITE(6,3) (A(1,J),A(2,J),J=1,N)
C
C      DATA REDUCTION
C
       DO 25 J=2,N
       IK=J-1
       TEMP1=A(1,J)
       TEMP2=A(2,J)
       DO 30 I=1,IK
       BOU1=A(1,I)+0.01*A(1,I)
       BOU2=A(1,I)-0.01*A(1,I)
       IF(A(1,J).GT.BOU1) GO TO 30
       IF(A(1,J).GT.BOU2.AND.A(1,J).LT.BOU1) GO TO 40
       M=IK
```

```
  100 SUM3=SUM3+((A(6,J)-SUM1/SUM2)**2)*A(2,J)
      STDEV=(SQRT(SUM3/SUM))**0.33333
C
C     CALCULATE MASS MEDIAN
C
      DO 80 J=1,N
      IF(AVMAS.LT.A(5,J)) GO TO 90
   80 CONTINUE
      DRMASS=A(5,N)*3.14156/6.0
      WRITE(6,6)AVMASS,DRMASS
      GO TO 999
   90 SMED=(AVMAS-A(5,J-1))/(A(5,J)-A(5,J-1))*
     1(A(3,J)-A(3,J-1))+A(3,J-1)
      WRITE(6,7)
      WRITE(6,9)SMEAN,STDEV,AVMASS,SMED
  998 CONTINUE
  999 STOP
      END
//GO.SYSLOUT DD SYSOUT=A
//GO.FT06F001 DD SYSOUT=A
//GO.FT07F001 DD SYSOUT=B
//GO.SYSIN DD *
/*
```

```
//JMDROP    JOB (                       ),'GORDON HILL',
// MSGLEVEL=1,MSGCLASS=A,CLASS=K,TIME=(0,29)
//*MAIN LINES=10,CARDS=20,ORG=RM028
//*FORMAT PU,DDNAME=,DEST=LOCAL
// EXEC FORTXCG
//SYSPRINT DD SYSOUT=A
//SYSTEM DD SYSOUT=A
//FORT.SYSIN DD *
C
C      PROGRAM TO EVALUATE  MMD'S
C
      DIMENSION A(10,1001),CIRCLE(12)
    1 FORMAT(2F10.3)
    2 FORMAT('1     INPUT DATA FOR POSITIONS= ',12A1,' WHICH HAS ',
     1I4,' INPUT CARDS',//,8X,'STAIN SIZE',5X,'NO. OF DROPS',//)
    3 FORMAT(7X,F10.3,5X,F10.1)
    4 FORMAT('1  REDUCED DATA HAS ',I4,' SETS OF DATA',//,8X,
     1'STAIN SIZE',5X,'NO. OF DROPS',5X,'DROP SIZE',10X,'N',10X,
     2'LOGN',13X,'D**2',//)
    5 FORMAT(7X,F10.3,5X,F10.1,5X,F10.5,5X,F10.1,5X,E13.6,5X,F10.7)
    7 FORMAT('1     COEFFICIENTS FOR LOGN VS D',//)
    8 FORMAT(12A1)
    9 FORMAT( 6X,' INTERCEPT= ',F10.6,/,6X,'SLOPE= ',F10.6,/,
     16X,'REGRESSION COEFFICIENT= ',F10.6,/,6X,'M. MEAN D.= ',F10.6,
     2/,6X,'M. MEDIAN D.= ',F10.6,///)
   11 FORMAT(//,'      COEFFICIENTS FOR LOGN VS D**2',//)
      DO 998 NJK=1,1
      READ(5,8) (CIRCLE(I),I=1,12)
      DO 10 J=1,1001
      N=J-1
      READ(5,1)A(1,J),A(2,J)
      IF(A(1,J).EQ.0.0.AND.A(2,J).EQ.0.0)GO TO 20
   10 CONTINUE
   20 WRITE(6,2) (CIRCLE(I),I=1,12),N
      WRITE(6,3) (A(1,J),A(2,J),J=1,N)
C
C      DATA REDUCTION
C
      DO 25 J=2,N
      IK=J-1
      TEMP1=A(1,J)
      TEMP2=A(2,J)
      DO 30 I=1,IK
      BOU1=A(1,I)+0.01*A(1,I)
      BOU2=A(1,I)-0.01*A(1,I)
      IF(A(1,J).LT.BOU2) GO TO 30
      IF(A(1,J).GT.BOU2.AND.A(1,J).LT.BOU1) GO TO 40
      M=IK
```

```
   45 A(1,M+1)=A(1,M)
      A(2,M+1)=A(2,M)
      IF(M.EQ.I)  GO TO 47
      M=M-1
      GO TO 45
   47 A(1,I)=TEMP1
      A(2,I)=TEMP2
      GO TO 25
   40 A(2,I)=A(2,I)+A(2,J)
      A(2,J)=0.0
      GO TO 25
   30 CONTINUE
   25 CONTINUE
      IK=0
      DO 50 J=1,N
      IK=IK+1
      IF(A(2,J).EQ.0.0) GO TO 60
      A(1,IK)=A(1,J)
      A(2,IK)=A(2,J)
      GO TO 50
   60 IK=IK-1
   50 CONTINUE
      N=IK
C
C     EXPANDING THE MATRIX
C
      SUM=0.0
      SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      SUM4=0.0
      SUM5=0.0
      SUM6=0.0
      SUM7=0.0
      SUM8=0.0
      DO 70 J=1,N
      A(3,J)=0.192*(A(1,J))**0.997
      SUM=SUM+A(2,J)
      A(4,J)=SUM
      A(5,J)=ALOG10(A(4,J))
      A(6,J)=A(3,J)**2
      A(7,J)=A(6,J)**2
      A(8,J)=A(6,J)*A(5,J)
      A(9,J)=A(3,J)*A(5,J)
      A(10,J)=A(5,J)**2
      SUM1=SUM1+A(6,J)
      SUM2=SUM2+A(7,J)
      SUM3=SUM3+A(3,J)
```

```
      SUM4=SUM4+A(9,J)
      SUM5=SUM5+A(8,J)
      SUM6=SUM6+A(3,J)
      SUM7=SUM7+A(5,J)
      SUM8=SUM8+A(10,J)
   70 CONTINUE
      WRITE(6,4) N
      WRITE(6,5) (A(1,J),A(2,J),A(3,J),A(4,J),A(5,J), A(6,J),J=1,N)
      IF(N.EQ.1) GO TO 100
C
C     CALCULATE COEFFICIENTS FOR LOG N VS D
C
      A1=(SUM1*SUM7-SUM3*SUM4)/(N*SUM1-SUM3**2)
      B1=(N*SUM4-SUM3*SUM7)/(N*SUM1-SUM3**2)
      R1=(N*SUM4-SUM3*SUM7)/(SQRT((N*SUM1-SUM3**2)*(N*SUM8-SUM7**2)))
      DM1=0.789/ABS(B1)
      D01=1.594/ABS(B1)
C
C     CALCULATE COEFFICIENTS FOR LOG N VS D**2
C
      A2=(SUM2*SUM7-SUM1*SUM5)/(N*SUM2-SUM1**2)
      B2=(N*SUM5-SUM1*SUM7)/(N*SUM2-SUM1**2)
      R2=(N*SUM5-SUM1*SUM7)/(SQRT((N*SUM2-SUM1**2)*(N*SUM8-SUM7**2)))
      DM2=0.7245/SQRT(ABS(B2))
      D02=0.9733/SQRT(ABS(B2))
      GO TO 150
  100 A1=0.0
      A2=0.0
      B1=0.0
      B2=0.0
      R1=0.0
      R2=0.0
      DM1=A(3,1)
      DM2=A(3,1)
      D01=A(3,1)
      D02=A(3,1)
C
C     PRINT RESULTS
C
  150 WRITE(6,7)
      WRITE(6,9)A1,B1,R1,DM1,D01
      WRITE(6,11)
      WRITE(6,9)A2,B2,R2,DM2,D02
  998 CONTINUE
  999 STOP
      END
//GO.SYSLOUT DD SYSOUT=A
//GO.FT06F001 DD SYSOUT=A


//GO.FT07F001 DD SYSOUT=B
//GO.SYSIN DD *
/*
```

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Formulae are derived to calculate the mass median (D_0) and mass mean (D_M) diameters of droplets in sprays. These formulae are based on an inverse exponential relationship which is empirically observed to occur in liquid sprays. The median diameter divides the total mass of liquid in half while the mean diameter represents a more commonly used statistical average for identifying random samples. The formulae are used to describe the characteristics of a typical set of experimental drop data and are compared to conventional formulae for calculating these parameters. (U)

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